

Exam 2
BME 301

BME 301

- For all students: there are 5 questions to be answered for the basic grade.
- For 301001 and 301003 students, question 6 is for extra credit.
- For 301HM1 students, question 6 is mandatory and choose 4 for the basic grade and one more for extra credit.
- Write **neatly** and **clearly**.

Exam 2

1. For the ECG signal following
 - a. How would you determine the minimum sampling rate of any signal?
 - b. What is the process called to assure you get a better sampling rate?

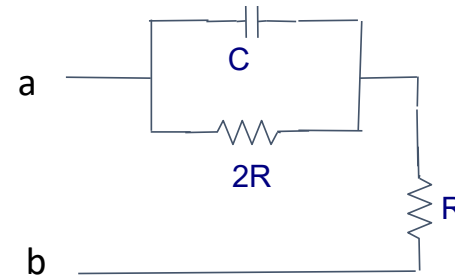
2. What is the most important part of the design process?

3. For an Arduino
 - a) For an Arduino Sketch, what are Global Variables for and where is it defined?
 - b) What is does the IDE stand for and how used in Sketch?

Exam 2

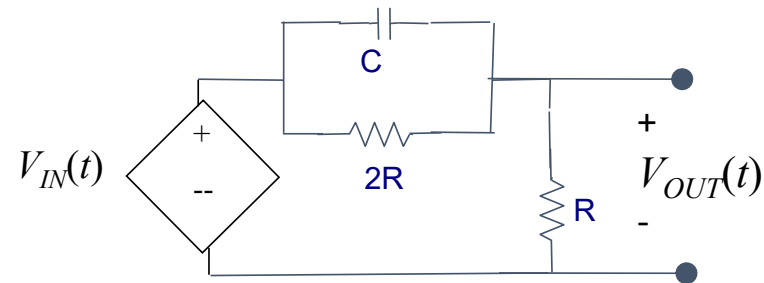
4. Impedances

- a) Find and sketch the **impedance** of the following circuit with respect to terminals a-b. in **polar form** and sketch the impedance as a function of **radian frequency, ω** , i.e. **in radians/second**. Assume $R=1$ and $C=2$



5. For the following circuit:

- a) Determine and sketch the transfer function in **polar form**.
b) Assume that $R=1$ and $C=2$, sketch the transfer function versus the **frequency, f** ; i.e. **in Hertz**.
c) What sort of circuit is this?
d) What is its cutoff frequency?



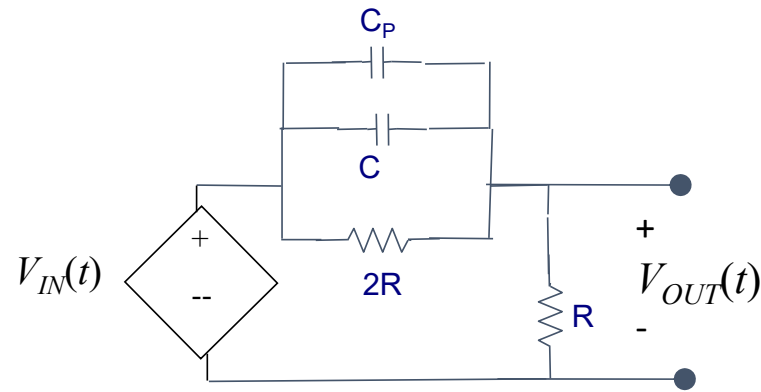
Exam 2

6. For Honors/extra credit

Repeat steps a-d of problem 5 for the following circuit.

Hint: Is there a way to simplify the circuit ?

$$C_p=3, C=2, R=1$$



Exam 2

1. For the ECG signal following

- a. How would you determine the minimum sampling rate of any single?

To determine the minimum sampling rate you need to determine the highest frequency of the identified signal and then multiply it by 2 to determine the minimum sampling rate.

- b. What is the process called to assure you get a better sampling rate?

Sample at a higher rate than the minimum sampling rate: multiply the minimum sampled rate by a number greater than zero, e.g. 10x. This is called Oversampling.

Homework

2. What is the most important part of the design process?

The Customer Needs are key to the Design Process. All device design decisions are made based on the Customer Needs.

Homework

3. For an Arduino

- a) For an Arduino Sketch, what are Global Variables for and where is it be defined?

Global Variables can be used for both the Setup and Loop sections of the Sketch and are defined in the Global Section which is located at the beginning of the Sketch; that is, before the Setup section or the Loop section is there is no Setup section.

- a) What is does the IDE stand for and how used in Sketch?
- i. IDE is know as the Integrated Development Environment (IDE).
 - ii. It supports program program development, verification, and downloading to the Arduino board.
 - iii. IDE support a serial monitor to allow users to interface with the program running on the Arduino board.
 - iv. An Arduino program is called a Sketch and is written in C/C++.

Exam 2

4. Impedances

- a) Find the **impedance** of the following circuit with respect to terminals a-b. in ***polar form*** and sketch the impedance as a function of ***radian frequency, ω*** , i.e. ***in radians/second***. Assume $R=1$ and $C=2$

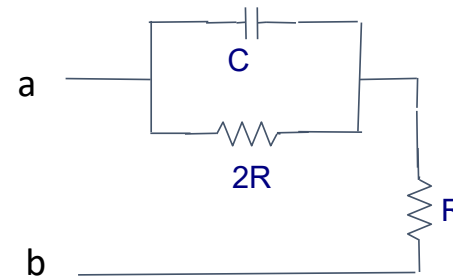
$$Z_{ab} = 2R \parallel Z_C + R$$

$$2R \parallel Z_C = \frac{2R \times \frac{1}{j\omega C}}{2R + \frac{1}{j\omega C}} = \frac{2R}{1 + j\omega 2RC}$$

$$Z_{ab} = 2R \parallel Z_C + R = \frac{2R}{1 + j\omega 2RC} + R = \frac{2R + R + j\omega 2R^2C}{1 + j\omega 2RC}$$

$$Z_{ab} = \frac{3R + j\omega 2R^2C}{1 + j\omega 2RC} = \frac{\sqrt{(3R)^2 + (\omega 2R^2C)^2} \angle \tan^{-1}\left(\frac{\omega 2R^2C}{3R}\right)}{\sqrt{1 + (\omega 2RC)^2} \angle \tan^{-1}(\omega 2RC)}$$

$$= \frac{\sqrt{(3R)^2 + (\omega 2R^2C)^2}}{\sqrt{1 + (\omega 2RC)^2}} \angle \tan^{-1}\left(\frac{2}{3}\omega RC\right) - \tan^{-1}(\omega 2RC)$$



Exam 2

4. Impedances

a) Find the **impedance** of the following circuit with respect to terminals a-b. in **polar form** and sketch the impedance as a function of **radian frequency, ω** , i.e. **in radians/second**. Assume $R=1$ and $C=2$

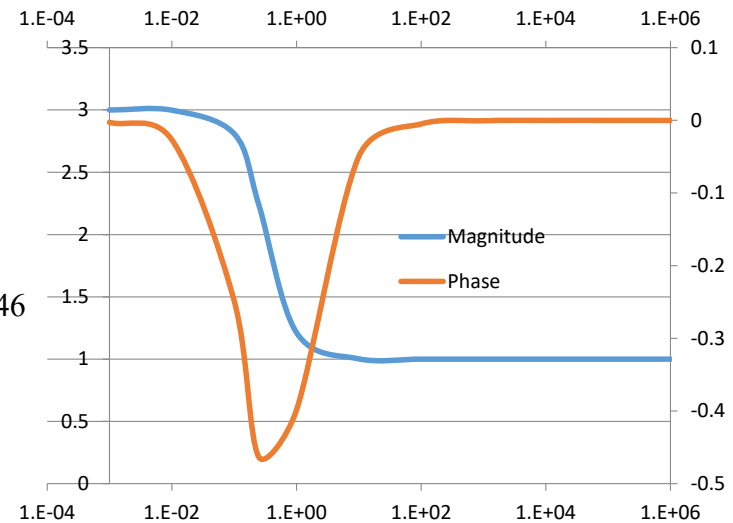
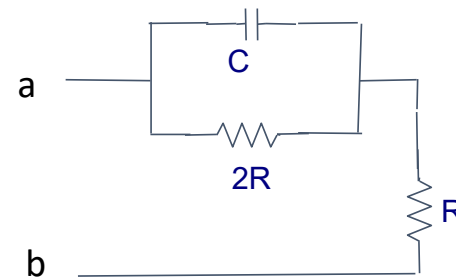
$$Z_{ab} = \frac{3R + j\omega 2R^2C}{1 + j\omega 2RC} = R \frac{3 + j\omega 2RC}{1 + j\omega 2RC}$$

$$Z_{ab} |_{\omega=0} = R \frac{3 + j0 2RC}{1 + j0 2RC} = 3R \angle 0 = 3 \angle 0$$

$$Z_{ab} |_{\omega \rightarrow \infty} \rightarrow R \frac{j\omega 2RC}{j\omega 2RC} = R \angle 0 = 1 \angle 0$$

$$Z_{ab} |_{\omega = \frac{1}{2RC} = \frac{1}{4}} = R \frac{3 + j \frac{1}{2RC} 2RC}{1 + j \frac{1}{2RC} 2RC} = R \frac{3 + j}{1 + j}$$

$$= R \frac{\sqrt{9+1} \angle \tan^{-1}(\frac{1}{3})}{\sqrt{2} \angle \frac{\pi}{4}} = R \frac{\sqrt{10}}{\sqrt{2}} \angle \tan^{-1}(\frac{1}{3}) - \frac{\pi}{4} = R\sqrt{5} \angle \tan^{-1}(\frac{1}{3}) - \frac{\pi}{4} = 2.24 \angle -0.46$$



Exam 2

5. For the following circuit:

- Determine the transfer function in ***polar form***
- Assume that $R=1$ and $C=2$, sketch the transfer function versus the ***frequency, f***; i.e. ***in Hertz***.
- What sort of circuit is this?
- What is its cutoff frequency?

$$a) \frac{V_{out}}{V_{in}} = \frac{R}{Z_{in}}$$

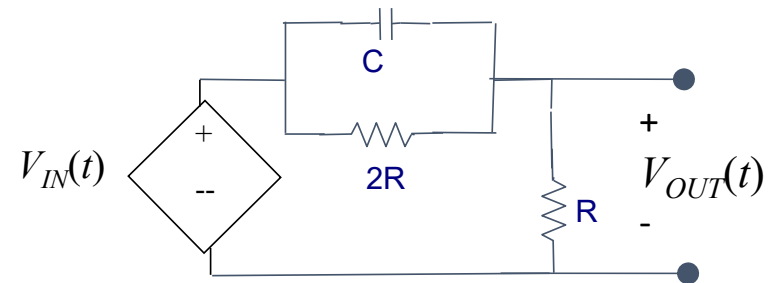
and $Z_{in} = Z_{ab}$ from problem 4; that is:

$$Z_{in} = Z_{ab} = R \frac{3 + j\omega 2RC}{1 + j\omega 2RC}$$

$$\frac{V_{out}}{V_{in}} = \frac{R}{R \frac{3 + j\omega 2RC}{1 + j\omega 2RC}} = \frac{R(1 + j\omega 2RC)}{R(3 + j\omega 2RC)} = \frac{1 + j\omega 2RC}{3 + j\omega 2RC}$$

Let $\omega = 2\pi f$ and $f_o = \frac{1}{2\pi 2RC} = 0.039$; then

$$\frac{V_{out}}{V_{in}} = \frac{1 + j \frac{f}{f_o}}{3 + j \frac{f}{f_o}} = \frac{\sqrt{1 + (\frac{f}{f_o})^2} \angle \tan^{-1}(\frac{f}{f_o})}{\sqrt{3^2 + (\frac{f}{f_o})^2} \angle \tan^{-1}(\frac{f}{3f_o})} = \frac{\sqrt{1 + (\frac{f}{f_o})^2}}{\sqrt{3^2 + (\frac{f}{f_o})^2}} \angle \tan^{-1}(\frac{f}{f_o}) - \angle \tan^{-1}(\frac{f}{3f_o})$$



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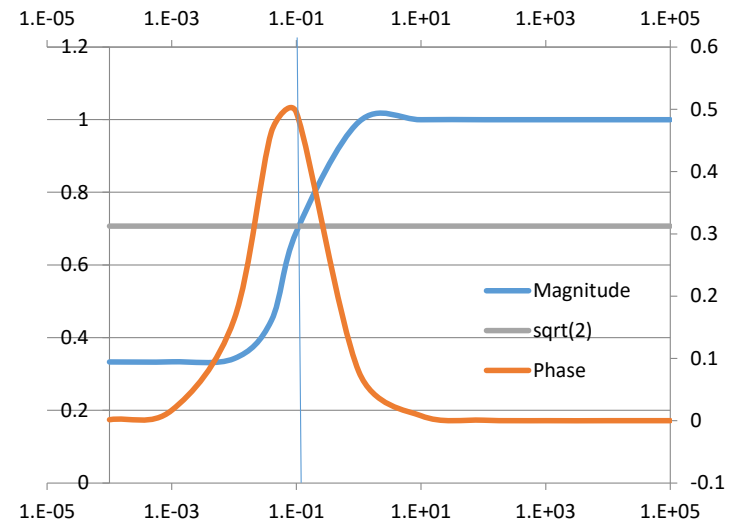
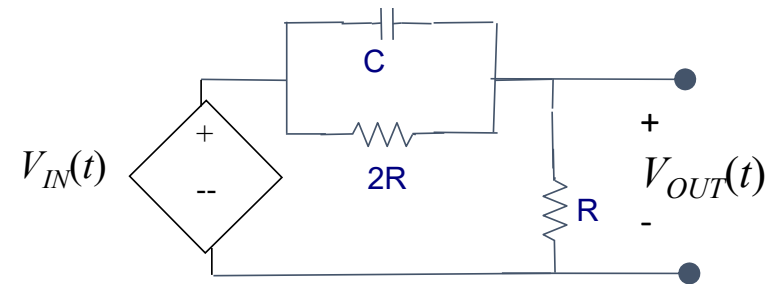
c) It may look like a HPF; however, it is not a filter since there is no stop band. That is there is no section of the transfer function where it has a value of zero.

$$b) \frac{V_{out}}{V_{in}} = \frac{1 + j \frac{f}{f_o}}{3 + j \frac{f}{f_o}}$$

$$\frac{V_{out}}{V_{in}} \Big|_{f=0} = \frac{1 + j \frac{0}{f_o}}{3 + j \frac{0}{f_o}} = \frac{1}{3} \angle 0$$

$$\frac{V_{out}}{V_{in}} \Big|_{f \rightarrow \infty} \rightarrow \frac{1 + j \frac{f}{f_o}}{3 + j \frac{f}{f_o}} \Big|_{f \rightarrow \infty} \rightarrow \frac{j \frac{f}{f_o}}{j \frac{f}{f_o}} = 1 \angle 0$$

$$\frac{V_{out}}{V_{in}} \Big|_{f=f_o=0.039} = \frac{1 + j \frac{f_o}{f_o}}{3 + j \frac{f_o}{f_o}} = \frac{1 + j1}{3 + j1} = \frac{\sqrt{2} \angle \frac{\pi}{4}}{\sqrt{10} \angle \tan^{-1}(\frac{1}{3})} = \frac{1}{\sqrt{5}} \angle \frac{\pi}{4} - \tan^{-1}(\frac{1}{3}) = 0.045 \angle 0.46$$



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- Determine the transfer function in **polar form**
- Assume that $R=1$ and $C=2$, sketch the transfer function versus the **frequency, f** ; i.e. **in Hertz**.
- What sort of circuit is this?
- What is its cutoff frequency?

d) from the graph the cutoff frequency is around $f = .1$ Hz; Note that f_o is not the cutoff frequency

since the value of $\frac{V_{out}}{V_{in}}$ is not equal $\frac{1}{\sqrt{2}} \times \frac{V_{out}}{V_{in}}|_{max} = \frac{1}{\sqrt{2}} = 0.707$ but equal to 0.45.

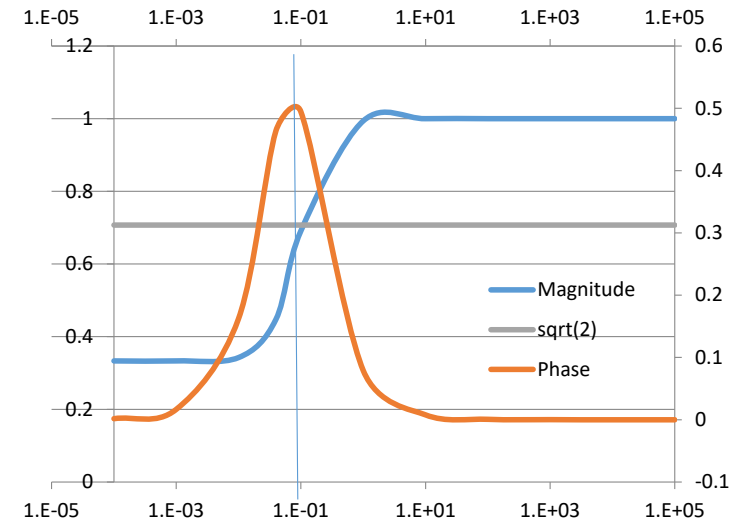
Although this is not really a filter, the definition of the cutoff frequency, f_{co} , is that

frequency when the magnitude of the transfer function is shown and must be equal to $\frac{1}{\sqrt{2}}$:

$$\left| \frac{V_{out}}{V_{in}} \right| = \left| \frac{1 + j \frac{f}{f_o}}{3 + j \frac{f}{f_o}} \right| = \frac{\sqrt{1 + \left(\frac{f_{co}}{f_o}\right)^2}}{\sqrt{3^2 + \left(\frac{f_{co}}{f_o}\right)^2}} = \frac{1}{\sqrt{2}} \quad \text{OR} \quad \frac{1 + \left(\frac{f_{co}}{f_o}\right)^2}{3^2 + \left(\frac{f_{co}}{f_o}\right)^2} = \frac{1}{2}$$

$$2\left(1 + \left(\frac{f_{co}}{f_o}\right)^2\right) = 9 + \left(\frac{f_{co}}{f_o}\right)^2 \Rightarrow \left(\frac{f_{co}}{f_o}\right)^2 = 7 \Rightarrow f_{co} = \sqrt{7} \times f_o$$

$$= \sqrt{7} \times \frac{1}{2\pi \times 2RC} = 2.65 \times 0.039 = 0.105 \text{ Hz}$$

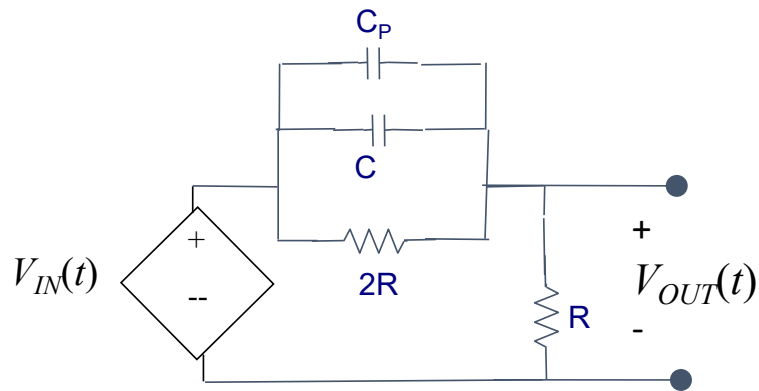


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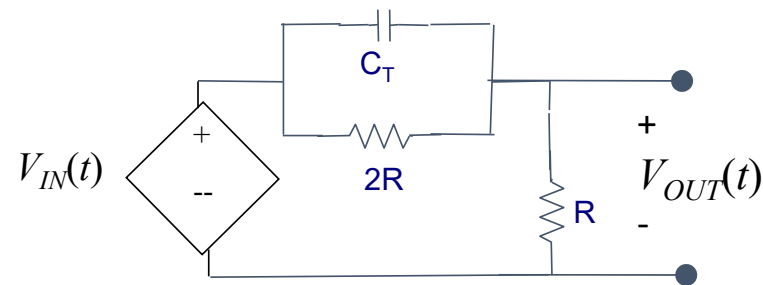
6. For HONORS and extra credit

Repeat steps a-d of problem 5 for the following circuit. Hint: Is there a way to simplify the circuit ?

$C_p=3$, $C=2$, $R=1$



Since C and C_p are in parallel, let's replace it by C_T for the total Capacitance and work with this circuit. Note that C_T is equal to $5f$.

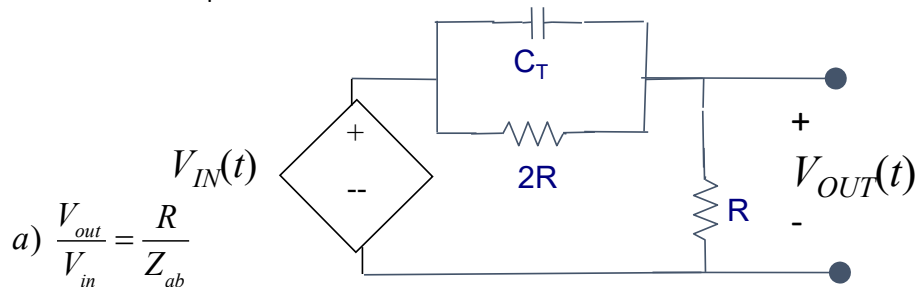


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Repeat steps a-d of problem 5 for the following circuit. Hint: Is there a way to simplify the circuit ?

$C_p=3, C=2, R=1$



$$a) \frac{V_{out}}{V_{in}} = \frac{R}{Z_{ab}}$$

$$Z_{ab} = \frac{3R + j\omega 2R^2 C_T}{1 + j\omega 2RC_T}; \text{ using } Z_{ab} \text{ from problem 4 with } C \text{ replaced by } C_T$$

$$\frac{V_{out}}{V_{in}} = \frac{R}{\frac{3R + j\omega 2R^2 C_T}{1 + j\omega 2RC_T}} = \frac{(1 + j\omega 2RC_T)}{3 + j\omega 2RC_T}; \text{ OR using } \frac{V_{out}}{V_{in}} \text{ from problem 5 with } C \text{ replaced by } C_T$$

Let $\omega = 2\pi f$ and $f_o = \frac{1}{2\pi 2RC_T} = 0.0159$ (note that f_o has changed since the equivalent capacitance is now 5 farads instead of 2 farads) and

$$\frac{V_{out}}{V_{in}} = \frac{1 + j\frac{f}{f_o}}{3 + j\frac{f}{f_o}}$$

$$b) \frac{V_{out}}{V_{in}} = \frac{1 + j\frac{f}{f_o}}{3 + j\frac{f}{f_o}}; \frac{V_{out}}{V_{in}} \Big|_{f=0} = \frac{1 + j\frac{0}{f_o}}{3 + j\frac{0}{f_o}} = \frac{1}{3} \angle 0; \frac{V_{out}}{V_{in}} \Big|_{f \rightarrow \infty} \rightarrow \frac{1 + j\frac{f}{f_o}}{3 + j\frac{f}{f_o}} \Big|_{f \rightarrow \infty} \rightarrow \frac{j\frac{f}{f_o}}{j\frac{f}{f_o}} = 1 \angle 0$$

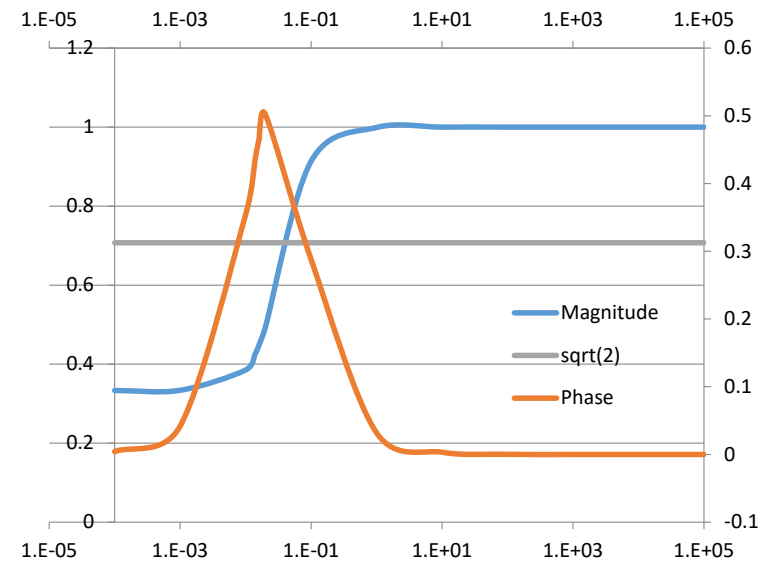
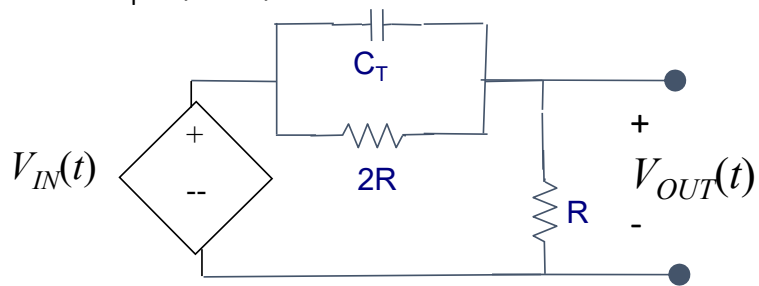
$$\frac{V_{out}}{V_{in}} \Big|_{f=f_o=0.0159} = \frac{1 + j\frac{f_o}{f_o}}{3 + j\frac{f_o}{f_o}} = \frac{1 + j1}{3 + j1} = \frac{\sqrt{2} \angle \frac{\pi}{4}}{\sqrt{10} \angle \tan^{-1}(\frac{1}{3})} = \frac{1}{\sqrt{5}} \angle \frac{\pi}{4} - \tan^{-1}(\frac{1}{3}) = 0.045 \angle 0.46$$

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c) It may look like a HPF; however, it is not a filter since there is no stop band.

That is there is no section of the transfer function where it has a value of zero.

6. For HONORS and extra credit

Exam 2

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$C_p=3, C=2, R=1$

d) from the graph the cutoff frequency is around $f = 0.05$ Hz; Note that f_o is not the cutoff frequency

since the value of $\frac{V_{out}}{V_{in}}$ is not equal $\frac{1}{\sqrt{2}} \times \frac{V_{out}}{V_{in}}|_{\max} = \frac{1}{\sqrt{2}} = 0.707$ but equal to 0.45.

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$$= \sqrt{7} \times \frac{1}{2\pi \times 2RC} = 2.65 \times 0.0159 = 0.042 \text{ Hz}$$

